

## GEOMETRY AND MEASUREMENT CONTENT KNOWLEDGE OF PRESERVICE K-8 MATHEMATICS TEACHERS: A SYNTHESIS OF RESEARCH

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*In this paper, we summarize the findings of a research synthesis specifically related to the geometry and measurement content knowledge of elementary/middle school preservice teachers. Findings from this synthesis show that preservice teachers have weak conceptions in geometry and measurement content knowledge. However, instructional strategies that incorporate technology (e.g., dynamic geometry software or virtual manipulatives) or analyzing children's work, have been shown to strengthen preservice teachers' understanding.*

### Introduction

For the past several years, a working group has met at the Psychology of Mathematics Education-North American Chapter (PME-NA) annual conferences. With members sharing their individual research, common themes and interests emerged and discussions turned towards needs for future work. In order for the group to build on and extend the knowledge base in the area of mathematical content knowledge for preservice teachers, a common need that emerged was knowledge of the status of current related research. Thus, a synthesis of the current research was essential for the following reasons:

1. To design a program of study to develop elementary/middle school preservice teachers' mathematical content knowledge for teaching, we need to understand *what* content knowledge preservice teachers bring to their courses. "The key to turning even poorly prepared prospective elementary teachers into mathematical thinkers is to work from what they *do* know" (CBMS, 2001, p. 17).
2. In addition to knowing *what* knowledge the preservice teachers bring to their coursework, we need to understand *how* to build on that knowledge in meaningful ways to further develop the preservice teachers' mathematical content knowledge for teaching. We need more empirical evidence of what learning opportunities contribute most to more knowledgeable and confident teachers in order to make more informed changes to our preparation programs (Mewborn, 2000).
3. To identify future research directions, we need to know *what* has been established in terms of preservice teachers' content knowledge and *how* that knowledge has been developed and *what* we still do not know or understand.

### Theoretical Framework

There has been a recent emphasis in the mathematics education community in describing the needed and desired mathematical content knowledge for teaching, with various descriptions emerging from research (e.g. Hill, Rowan, & Ball, 2005; Ma, 1999; National Research Council, 2001; Shulman, 1986). Building on the work of Shulman (1986), Hill, Ball, & Shilling (2008) describe a framework for distinguishing different types of knowledge included in a construct of

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mathematical knowledge for teaching. This framework distinguishes between subject matter knowledge and pedagogical content knowledge. The interest of the working group centered on synthesizing research on subject matter (content) knowledge described in this framework. Understanding this content knowledge is vital for mathematics educators as we develop content (and methods) courses to meaningfully prepare preservice teachers as well as knowing *how* to help preservice teachers understand the needed content knowledge for themselves.

Our guiding questions for this synthesis were: *What research has been conducted on elementary/middle school preservice teachers' content knowledge?* and *What is known from this research about preservice teachers' content knowledge?* This paper reports on the work related to the subgroup focusing on preservice teachers' geometry and measurement content knowledge.

## Methods

We began our work using the ERIC database focusing on the years 1998 to 2009. For dissertation work, we chose to review those from 2006 to 2009 and conference proceedings 2004-2009. In addition to the publication date requirement, each research study had to be published in a peer-reviewed journal and focus on elementary or middle school preservice teachers' geometry and measurement content knowledge. Since not all countries use the same grade-level classification as in the US, we decided to look at children aged 3 - 12 (possibly up to 14) when including international studies. Key words used in various combinations during this search process included geometry, preservice/pre-service, elementary, teacher, mathematics, measurement, length, volume, area, and angle. This produced a total of 30 studies.

Each of the 30 studies went through an independent review that detailed the research questions, study type and research design, lens and/or approach used, selection and description of participants, conditions of and procedures for data collection, data analysis, findings, and conclusions/implications. When there was a discrepancy on the inclusion or exclusion of a study, discussion allowed for mutual consensus on a final decision. Excluded from our synthesis were studies that had: (a) a general description of content knowledge that lacked specific attention to geometry or measurement, (b) a selection of inservice teachers or college students majoring in mathematics as opposed to mathematics education, (c) a sole focus on perceptions about mathematics not connected to content knowledge needed for teaching, and (d) a focus on describing classroom practice or activities with a lack of attention to research design methods. After examining the 30 studies, only 11 met all of the review criteria.

When the PME-NA Working Group met again to discuss the status of all content area reviews, the group decided to conduct another search through the literature. This second phase of the search focused on the 25 journals that provided relevant research studies for any of the content areas during the first phase. This second phase was initiated to search for relevant research published in 2010 and to see if new studies surfaced that possibly were missed in the initial search. Two new studies were found for geometry and measurement; one was published in 2010 and the second was published in a year not included in the ERIC database for that particular journal. Thus, in our final analysis, only 13 studies matched our criteria to be included in our analysis of research describing elementary preservice teachers' knowledge of geometry and measurement.

## Results

### *Characterizations of the data*

Eleven studies focused on geometry and only two studies reported on measurement. Of the geometry studies, seven focused on preservice teachers' content knowledge relative to shapes

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and their attributes (diagonals, reflective symmetry, etc.), one examined visualization skills, and four focused on other general knowledge of geometry. Studies on shape focused on quadrilaterals, rectangles/rhombi, and triangles. Studies on measurement focused on perimeter, area, and volume.

In order to synthesize findings across the 13 studies, we examined study types and characterized the research questions that dealt with preservice teachers' geometry and measurement content knowledge. Three broad categories emerged: descriptive studies that examined the *status* of mathematical knowledge and understandings of preservice teachers; studies that *explored* the impact of some "treatment" in a mathematics content or methods course; and finally, comparison studies that examined *differences* between ideas/features, groups or looked for relationships between two entities. Italicized text emphasizes the key features of the questions in the studies for these three categories.

#### *The Status of Preservice Teachers' Content Knowledge in Geometry and Measurement*

Six studies had at least one research question that focused on what preservice teachers understand about specific topics in geometry and measurement. These topics included quadrilaterals, reflective symmetry, triangle altitudes, volume, perimeter, and area.

Across all of these studies, we found preservice teachers' understanding of geometry to be weak or limited. "Preservice teachers' understanding of geometry was limited with shapes and measurement aspects of shapes" (Aslan-Tutak, 2009, p. 158); "it was found that a large portion of preservice teachers has lack of content knowledge of reflective symmetry" (Son, 2006, p. 149); "the results presented in this study are that most of these participants lack the ability to articulate complete descriptions of rectangle and rhombus" (Pickreign, 2007, p. 6); "we found that many preservice teachers had the same poor concept images as primary or secondary students" (Gutierrez & Jaime, 1999, p. 272); "the diversity of responses offered by preservice teachers in this study could be comparable to what would be expected from students in an upper primary classroom" (Zevenbergen, 2005, p. 21); and "yet, even with an apparently better foundation in mathematics, the students appeared to have poor conceptual understanding (*in perimeter and area*)" (Menon, 1998, p. 365).

Pickreign (2007) attends to how teacher preparation programs might address the concern of such limited geometrical understanding of preservice teachers, very similar to our initial quote from CBMS, working from what preservice teachers do know:

"This, however, raises serious questions to be answered if we are to affect the content knowledge of teachers of mathematics: Can sufficient experience with these geometric ideas be provided in teacher education programs to lead to more profound understandings? Are there other mathematical ideas that require such experience? What should characterize these experiences? Can these experiences be provided without adding time or credit hours to teacher certification programs? One option in addressing these questions might be to not address them. Instead, accept teacher education programs as entry level preparation and utilize continuing professional development to address the growth of teachers' profound understanding of fundamental mathematics." (p. 6)

Time appears to be a critical component. Perhaps teacher preparation programs will not be able to provide the needed extent of time to make significant change but it can lay important groundwork.

#### *Exploring the Impact of a Treatment*

Five studies had questions of an exploratory nature, investigating "what happens if", four in *Wiest, L. R., & Lambreg, T. (Eds.). (2011). Proceedings of the 33rd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Reno, NV: University of Nevada, Reno.*

the area of geometry and one looking at volume.

Geometry studies explored how dynamic geometry softwares impact preservice teachers' understanding of proof, the influence of the analysis of children's work with quadrilaterals on preservice teachers' understanding of geometry content knowledge, the impact of instruction utilizing graphic organizers and concept attainment strategies on preservice teachers' understanding of altitudes of triangles and diagonals of polygons, and the influence of digital and concrete tangrams on preservice teachers' 2D visualization on tangram designs.

Findings from Christou, Mousoulides, Pittalis, and Pitta-Pantazi (2004) show dynamic geometry softwares, along with appropriate questioning, motivated the students in the study to justify their conjectures and enabled students to pass from "exploratory" geometry to deductive geometry. Aslan-Tutak (2009) found that incorporating analysis of children's work into activities while preservice teachers make sense of quadrilaterals had a significant positive affect on the preservice teachers' learning as determined by a repeated measures ANOVA. Cunningham & Roberts (2010) explored the impact of a treatment lesson involving instructional strategies designed to assist the development of preservice teachers' concept images and concept definitions related to altitudes of triangles and diagonals of polygons. These strategies included the use of graphic organizers along with the concept attainment model in the development of definitions. The combination of these strategies resulted in some improvement for the preservice teachers' understanding of triangle altitudes and diagonals of polygons. Spencer's work (2008) found that using both digital and concrete tangrams in a methods course did help improve 2D visualization skills of preservice teachers which, in turn, had a significant positive impact on their attitude toward geometry.

Zevenbergen's (2005) study explored the impact of various learning dispositions that were emphasized within a mathematics course module on volume. These dispositions included developing mathematical meaning of volume as opposed to only using algorithmic methods, emphasizing the development of measurement and spatial sense, and developing the capacity within the preservice teachers to identify errors in children's mathematical thinking. However, despite the various methods used in the course to develop these dispositions, there was a "worrying number of students" who had not achieved them (p. 21), with some students quite resistant to alter their thinking about how to learn mathematics. Responses to interviews of students in the course highlighted the power of the teaching practicum, with preservice teachers rejecting the nature of the work done in their mathematics course due to their experiences in the schools. "Ideally, it would be useful to expose students to schools and classrooms that demonstrate the values embedded within teacher education courses if such courses are to effectively change teaching practice" (p. 21).

These few studies suggest various instructional strategies can make positive change on preservice teachers' geometrical understanding, such as those incorporating technology, making use of graphic organizers and concept attainment, and analyzing children's work. Field work that provides similar experiences as found in preservice teachers' mathematics courses and the length of time devoted to concept development may be critical factors to maintain these positive changes.

### *Examining Associations and Differences*

Six studies included research questions that examined relationships and/or differences. Studies examined differences in geometric reasoning levels between elementary and secondary preservice teachers and male and female preservice teachers, differences in content knowledge

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based on instruction, relationships between visualization skills and attitude towards geometry, differences in student learning environment, and differences in students' conceptions of geometry concepts.

Halat (2008) administered a *van Hiele Geometry Test (VHGT)*, based upon work of Usiskin (1982), to 281 Turkish elementary and secondary preservice teachers. He found "no statistically significant difference in regard to the reasoning stages between the pre-service elementary school and secondary mathematics teachers, and that although there was a difference with reference to van Hiele levels between male and female pre-service secondary mathematics teachers favoring males, there was no sex-related difference found between male and female pre-service elementary school teachers" (Halat, 2008, p. 1).

With respect to examining the impact of instructional strategies, Aslan-Tutak (2009) considered the impact of having supplemental geometry activities in a two-week unit during a methods courses, using control and treatment groups. The results showed the geometry knowledge of students, as measured by the *Content Knowledge for Teaching Mathematics (CKT-M)* test (Hill & Ball, 2004), in either group was increased significantly, however the grouping did not have any statistically significant affect on participants' knowledge growth. Lundin (2007) also used the *CKT-M* test to measure content knowledge differences of preservice teachers from two groups: those who had completed mathematics courses designed for elementary education majors and those preservice teachers who had completed the mathematics requirement through the completion of other mathematics courses. She found no statistically significant differences between the two groups based upon students' mean scores. Lundin suggests the lack of difference could be attributed "to the likelihood that mathematical content gained in regular math courses taken by the participants was as beneficial as the content knowledge gained in the courses examined in this study" (p. 78) or that the subjects in her study "did not make a transition from applying general mathematical content knowledge to the mathematical content knowledge required for teaching" (p. 79).

Spencer (2008) found a positive relationship between attitude toward geometry and 2D visualization with increased levels of 2D visualization corresponding to an increased attitude towards geometry. Two-dimensional visualization skills were improved by the use of concrete and/or digital tangrams.

Gerretson (2004) examined to see whether or not there was a difference in elementary preservice teachers' performance on similarity tasks when using dynamic geometry software as compared to paper-and-pencil learning environment using traditional tools (e.g., compass, ruler). Using a pre/posttest control group experiment using randomized blocks controlling for initial performance, she found a statistically significant difference in learning environments between the two treatment groups. "Fundamentally, software users outperformed non-software users even when prior knowledge variability was taken into consideration." (p. 18). Analysis suggests elementary preservice teachers using a paper-and-pencil learning environment encountered more difficulties particularly situated around similarity properties of unfamiliar shapes, whereas preservice teachers using dynamic geometry software had "acquired a greater knowledge base to access, network, and apply" (p. 19).

Fujita and Jones (2006) explored the nature of preservice teachers' *personal figural concepts* and *formal figural concepts*, particularly in the area of quadrilaterals, building on the work of Tall & Vinner (1981) with respect to concept image and concept definition and Fischbein's figural concept (1993). The study examined data from two groups of preservice teachers in Scotland. The groups were based on their year of study in the teacher education program; either

first or third. For either group, the data indicate a gap exists between the *formal figural concepts* of preservice teachers and their *personal figural concepts* “such that their images are so influential in their personal figural concepts that they dominate their attempt to define basic quadrilaterals” (Fujita & Jones, 2006, p. 131). Preservice teachers rely on their images of shapes to construct definitions rather than examining and using properties of shapes.

In several of these comparison studies, despite further evidence of preservice teachers’ limited conceptual understandings in geometry, growth in content knowledge was demonstrated. Growth occurred where technology was incorporated into the learning environment and when supplemental learning activities examining children’s work were utilized.

### Discussion and Conclusion

Our synthesis was guided by two questions, determining what research has been done related to geometry and measurement content knowledge and synthesizing what we know from this research regarding preservice teachers content knowledge. We organized the research found into three categories; briefly, these were status, explorations, and comparisons.

Based upon the research examining the status of preservice teachers’ knowledge of geometry content, there is a need for these courses to take the time to develop foundational knowledge of geometry and measurement.

Preservice teachers enter their teacher preparation programs with limited geometry and measurement content knowledge, relying more on their understandings from middle school than their high school geometry course and operating on low van Hiele levels of understanding, relying on their concept images of geometrical concepts, with poorly constructed concept definitions. Although one study found no significant difference in mean scores between preservice teachers who had taken mathematics courses designed for teachers and those who had not, others had positive findings. Studies that explored alternative methods of instruction with the use of technology, such as dynamic geometry systems and virtual manipulatives, provided encouraging results related to improving deductive thinking and 2D visualization skills. Other instructional strategies in this synthesis that promoted preservice teachers’ learning of shape and their properties included using a graphic organizer and concept attainment model, and analyzing children’s work. The importance of a “supportive” field component was also noted to help sustain the values of the teacher education program, otherwise it is likely preservice teachers would reject constructive methods of teaching mathematics.

Limitations to our synthesis include using only one database. Using multiple sources may have produced more studies. Expanding our search to include more than the previous ten years would also have included more research and perhaps more seminal pieces in the area of geometry and measurement.

This summative look at research on preservice teachers’ geometry and measurement content knowledge shows a paucity of research published in peer-reviewed arenas in the past ten years. However, this limited research consistently shows that preservice teachers do not have a solid understanding of several fundamental concepts within geometry and measurement. In order to work from what preservice teachers *do* know (CBMS, 2001), it appears that a greater focus on developing strong foundational knowledge is needed in geometry and measurement courses. Yet this raises some questions:

- What do we perceive, as a community of mathematics educators, as the essential foundational geometry and measurement knowledge that content courses should include?
- If such “core” foundational knowledge can be determined, what are sufficient

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experiences that would enable preservice teachers to acquire profound understandings of this core knowledge and when do these experiences occur in a teacher preparation program?

Further work of researchers mapping out the core mathematical content knowledge for teachers will be helpful in highlighting the gaps between what content knowledge is *needed* by preservice teachers and what knowledge they bring with them to their teacher preparation programs.

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